Ramanujan School of Mathematics

Class Test on Calculus, June 2019

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Total marks: 40 Time: 2 hours.

Attempt as many as you can. Answers without proper explanations will fetch zero. You can use any result proved in the class, but you have to state it properly.

1. (10 marks) Define $x_n = \sin a + \sin(a+d) + \sin(a+2d) + \cdots + \sin(a+(n-1)d)$, for $n \geq 1$. Find all real numbers a, d for which the sequence $\{x_n\}_{n\geq 1}$ is bounded.

2. (5+5 marks) Suppose that $\{x_n\}_{n\geq 1}$ and $\{y_n\}_{n\geq 1}$ are two convergent sequences, with $\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n$. Determine (with proof/counter-example) whether the following statements are true or false:

   (a) $\lim_{n\to\infty} (x_1 + \cdots + x_n) = \lim_{n\to\infty} (y_1 + \cdots + y_n)$.

   (b) $\lim_{n\to\infty} (x_n)^n = \lim_{n\to\infty} (y_n)^n$.

   (Note, a statement is false if it fails to hold even for just one case.)

3. (10 marks) Suppose that x_n satisfies $x_{n+1} = \sqrt{6 + x_n}$ for every $n \geq 1$, and let $x_1 = \sqrt{6}$. Show that x_n converges and also find the limit.

4. (10 marks) Suppose a is a positive real number. Define a sequence $\{x_n\}_{n\geq 1}$ by

$$x_n = \frac{[a] + [2a] + \cdots + [na]}{n^2}, n \geq 1.$$

   Prove that $\lim_{n\to\infty} x_n$ exists and also find the limit. (Here $[t]$ denotes the greatest integer less than or equal to t.)

Do not cheat to yourself. All the best!

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