Ramanujan School of Mathematics

Class Test on Calculus

Sept 2019

Total marks: $10 \times 5 = 50$ Time: 2 hours.

Attempt all the questions. Answers without proper explanations will fetch zero. Show all your rough work – partial solutions may be rewarded. You can use any theorem/result without proving it again; but you have to state it properly.

1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a continuous function such that $f(x) \neq x$ for every $x \in \mathbb{R}$. Is it possible that there exists some $c \in \mathbb{R}$ such that $f(f(c)) = c$?

2. Let $f : [0, 1] \to \mathbb{R}$ be a function satisfying $f(2x) = 3f(x)$ for every $0 \leq x \leq 1/2$. If f is bounded, show that $\lim_{x \to 0+} f(x) = f(0)$.

3. Determine, with proof, whether the following statements are true or false: (If true then provide a proof, else provide a counter-example)
   (a) If $\lim_{x \to 0} f(x) = c$ then $\lim_{x \to 0} f(\sin x) = c$.
   (b) If $\lim_{x \to 0} f(\sin x) = c$ then $\lim_{x \to 0} f(x) = c$.

4. Determine, with proof, the value of the following limit
$$\lim_{n \to \infty} \tan^n \left(\frac{\pi}{4} + \frac{1}{n} \right).$$

5. Let $f, g : \mathbb{R} \to \mathbb{R}$ be continuous functions such that given any two points $x_1 < x_2$, there exists a point x_3 between x_1 and x_2 such that $f(x_3) = g(x_3)$. Show that $f(x) = g(x)$ for every $x \in \mathbb{R}$.

Do not cheat to yourself. All the best!

Teacher: Aditya Ghosh